UNIVERSITY OF RAJASTHAN
JAIPUR

SYLLABUS

M.A. /M.Sc. MATHEMATICS
Annual Scheme

Previous Examination 2020

Final Examination 2021

Dy. Registrar (Academic-I)
University of Rajasthan
Jaipur

M.A./M.Sc.(Previous) Mathematics Examination – 2020

Scheme of Examination : Annual Scheme
Note: Papers I to V are compulsory

Paper – I: Advanced Abstract Algebra
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Direct product of groups (External and Internal). Isomorphism theorems – Diamond isomorphism theorem, Butterfly Lemma, Conjugate classes (Excluding p-groups), Commutators, Derived subgroups, Normal series and Solvable groups, Composition series, Refinement theorem and Jordan-Holder theorem for infinite groups.

Unit 2: Sylow's theorems (without proof), Cauchy's theorem for finite abelian groups. Euclidean rings. Polynomial rings and irreducibility criteria. Linear transformation of vector spaces, Dual spaces, Dual basis and their properties, Dual maps, Annihilator.

Unit 3: Field theory – Extension fields, Algebraic and Transcendental extensions, Separable and inseparable extensions, Normal extensions. Splitting fields.
Galois theory – the elements of Galois theory, Automorphism of extensions, Fundamental theorem of Galois theory, Solutions of polynomial equations by radicals and Insolvability of general equation of degree five by radicals.

Unit 4: Matrices of a linear maps, Matrices of composition maps, Matrices of dual map, Eigen values, Eigen vectors, Rank and Nullity of linear maps and matrices, Invertible matrices, Similar matrices, Determinants of matrices and its computations, Characteristic polynomial, minimal polynomial and eigen values.

Unit 5: Real inner product space, Schwartzs inequality, Orthogonality, Bessel's inequality, Adjoint, Self adjoint linear transformations and matrices, Orthogonal linear transformation and matrices, Principal Áxis Theorem.

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Reference Books:

1. Deepak Chatterjee, Abstract Algebra, Prentice – Hall of India (PHI), New Delhi, 2004
2. N.S.Gopalkrishnan, University Algebra, New Age International, 1986.
3. Qazi Zameeruddin and Surjeet Singh, Modern Algebra, Vikas Publishing,2006
4. G.C.Sharma, Modern Algebra, Shivlal Agrawal & Co., Agra,1998.
5. Joseph A. Gallian, Contemporary Abstract Algebra (4th Ed.), Narosa Publishing House, 1999.
6. David S. Dummit and Richard M. Foote, Abstract Algebra (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd, Singapore, 2004.
7. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra (4th Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.
8. I.N. Herstein, Topics in Algebra (2nd edition), John Wiley & Sons, 2006.
9. Michael Artin, Algebra (2nd edition), Pearson Prentice Hall, 2011.

Paper – II: Real Analysis and Topology

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Algebra and algebras of sets, Algebras generated by a class of subsets, Borel sets, Lebesgue measure of sets of real numbers, Measurability and Measure of a set, Existence of Non-measurable sets, Measurable functions, Realization of non-negative measurable function as limit of an increasing sequence of simple functions, Structure of measurable functions, Convergence in measure, Egoroff's theorem.

Unit 2: Weierstrass's theorem on the approximation of continuous function by polynomials, Lebesgue integral of bounded measurable functions, Lebesgue theorem on the passage to the limit under the integral sign for bounded measurable functions. Summable functions, Space of square summable functions. Fourier series and coefficients, Parseval's identity, Riesz-Fisher Theorem.

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Unit 3: Lebesgue integration on R^2, Fubini's theorem. L^p-spaces, Holder-Minkowski inequalities. Completeness of L^p-spaces, Topological spaces, Subspaces, Open sets, Closed sets, Neighbourhood system, Bases and sub-bases.

Unit 4: Continuous mapping and Homeomorphism, Nets, Filters, Separation axioms (To, T_1, T_2, T_3, T_4). Product and Quotient spaces.

Unit 5: Compact and locally compact spaces. One point compactification theorem. Connected and Locally connected spaces, Continuity and Connectedness and Compactness.

Reference Books:

1. Shanti Narayan, A Course of Mathematical Analysis, S. Chand & Co., N.D., 1995.
2. S.C.Malik and Savita Arora, Mathematical Analysis, New Age International, 1992.
3. T. M. Apostol, Mathematical Anslysis, Narosa Publishing House, New Delhi, 1985.
4. R.R. Goldberg, Real Analysis, Oxford & IBH Publishing Co., New Delhi, 1970.
5. S. Lang, Undergraduate Analysis, Springer-Verlag, New York, 1983.
6. James R. Munkres, Topology, 2nd Edition, Pearson International, 2000.
7. J. Dugundji, Topology, Prentice-Hall of India, 1975.
8. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
9. Walter Rudin, Real and Complex Analysis, Tata McGraw-Hill Pub. Co. Ltd., 1986.
10. I.N. Natansen, Theory of Functions of a Real Variable, Fredrik Pub. Co., 1964.

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Paper – III: Differential Equations and Special Functions

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Non-linear ordinary differential equations of particular forms. Riccati's equation –General solution and the solution when one, two or three particular solutions are known. Total Differential equations. Partial differential equations of second order with variable co-efficients- Monge's method.

Unit 2: Classification of linear partial differential equation of second order, Canonical forms. Cauchy's problem for first order partial differential equations, Method of separation of variables, Laplace, Wave and diffusion equations, Linear homogeneous boundary value problems. Eigen values and eigen functions. Strum-Liouville boundary value problems. Orthogonality of eigen functions. Reality of eigen values.

Unit 3: Calculus of variation – Functionals, Variation of a functional and its properties, Variational problems with fixed boundaries, Euler's equation, Extremals, Functional dependent on several unknown functions and their first order derivatives, Functionals dependent on higher order derivatives, Functionals dependent on the function of more than one independent variable. Variational problems in parametric form, Series solution of a second order linear differential equation near a regular singular point (Method of Frobenius) for different cases.

Unit 4: Gauss hypergeometric function and its properties, Integral representation, Linear transformation formulas, Contiguous function relations, Differentiation formulae, Linear relation between the solutions of Gauss hypergeometric equation, Kummer's confluent hypergeometric function and its properties, Integral representation, Kummer's first transformation. Legendre polynomials and functions $P_n(x)$ and $Q_n(x)$.

Unit 5: Bessel functions $J_n(x)$, Hermite polynomials $H_n(x)$, Laguerre and Associated Laguerre polynomials.

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Reference Books:

1. J.L.Bansal and H.S.Dhami, Differential Equations Vol-II, JPH, 2004.
2. M.D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Co., 2003.
3. L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, AMS, 1999.
4. I.N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1988.
5. E.A. Codington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, 1961.
6. Frank Ayres, Theory and Problems of Differential equations, TMH, 1990.
7. D.A. Murray, Introductory Course on Differential Equations, Orient Longman,1902.
8. A.R.Forsyth, A Treatise on Differential Equations, Macmillan & Co. Ltd., London, 1956.

Paper- IV: Differential Geometry and Tensor Analysis

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Space curves, Tangenet, Contact of curve and surface, Osculating plane, Principal normal and Binormal, Curvature, Torsion, Serret-Frenet's formulae, Osculating circle and Osculating sphere, Existence and Uniqueness theorems, Bertrand curves, Involute, Evolutes.

Unit 2: Ruled surface, Developable surface, Tangent plane to a ruled surface. Necessary and sufficient condition that a surface $\zeta = f(\xi,\eta)$ should represent a developable surface. Conoids, Inflexional tangents, Singular points, Indicatrix. Metric of a surface, First, second and third fundamental forms, Weingarten equations. Fundamental magnitudes of some important surfaces, Orthogonal trajectories.

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Unit 3: Normal curvature, Meunier's theorem. Principal directions and Principal curvatures, First curvature, Mean curvature, Gaussian curvature, Umbilics. Radius of curvature of any normal section at an umbilic on z = f(x,y). Radius of curvature of a given section through any point on z = f(x,y). Lines of curvature, Principal radii, Relation between fundamental forms. Asymptotic lines, Differential equation of an asymptotic line, Curvature and Torsion of an asymptotic line. Gauss's formulae, Gauss's characteristic equation, Mainardi-Codazzi equations. Fundamental existence theorem for surfaces, Parallel surfaces, Gaussian and mean curvature for a parallel surface, Bonnet's theorem on parallel surfaces.

Unit 4: Geodesics, Differential equation of a geodesic, Single differential equation of a geodesic, Geodesic on a surface of revolution, Geodesic curvature and Torsion, Normal angle, Gauss-Bonnet Theorem.
Tensor Analysis– Kronecker delta. Contravariant and Covariant tensors, Symmetric tensors, Quotient law of tensors, Relative tensor. Riemannian space. Metric tensor, Indicator, Permutation symbols and Permutation tensors.

Unit 5: Christoffel symbols and their properties, Covariant differentiation of tensors. Ricci's theorem, Intrinsic derivative, Geodesics, Differential equation of geodesic, Geodesic coordinates, Field of parallel vectors, Reimann-Christoffel tensor and its properties. Covariant curvature tensor, Einstein space. Bianchi's identity. Einstein tensor, Flate space, Isotropic point, Schur's theorem.

Reference Books:

1. R.J.T. Bell, Elementary Treatise on Co-ordinate geometry of three dimensions, Macmillan India Ltd., 1994.
2. Mittal and Agarwal, Differential Geometry, Krishna publication, 2014.
3. Barry Spain, Tensor Calculus, Radha Publ. House Calcutta,1988.
4. J.A. Thorpe, Introduction to Differential Geometry, Springer-Verlog , 2013.
5. T.J. Willmore - An Introduction to Differential Geometry. Oxford University Press.1972.
6. Weatherbum, Reimanian Geometry and Tensor Clculus, Cambridge Univ. Press, 2008.
7. Thorpe, Elementary Topics in Differential Geometry, Springer Verlag, N.Y. (1985).
8. R.S. Millman and G.D. Parker, Elements of Differential Geometry, Prentice Hall, 1977.

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Paper – V: Mechanics

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: D'Alembert's Principle. General equations of motion of a rigid body. Motion of the centre of inertia and motion relative to the centre of inertia. Motion about a fixed axis. The compound pendulum, Centre of percussion. Conservation of momentum (linear and angular) and energy for finite as well as impulsive forces.

Unit 2: Motion in three dimensions with reference to Euler's dynamical and geometrical equations. Motion under no forces, Motion under impulsive forces. Motion of a Top.

Unit 3: Lagrange's equations for holonomous dynamical system, Energy equation for conservative field, Small oscillations, Motion under impulsive forces. Hamilton's equations of motion, conservation of energy, Hamilton's Principle and Principle of Least Action.

Unit 4: Kinematics of ideal fluids. Lagrange's and Euler's methods. Equation of continuity in Cartesian, cylindrical and spherical polar coordinates. Boundary surface. Stream-lines, path-lines ,velocity potential, rotational and irrotational motion.

Unit 5: Euler's hydrodynamical equations. Bernoulli's theorem. Helmholtz equations. Cauchy's integrals, Motion due to impulsive forces. Motion in two-dimensions: Stream function, Complex potential. Sources, Sinks, Doublets, Images in two-dimensions.

Reference Books:

1. N. C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw-Hill, 1991.
2. M. Ray and H.S. Sharma, A Text Book of Dynamics of a Rigid Body, Students' Friends & Co., Agra, 1984.
3. M.D. Raisinghania, Hydrodynamics, S. Chand & Co. Ltd., N.D. 1995.
4. M. Ray and G.C. Chadda, A Text Book on Hydrodynamics, Students' Friends & Co., Agra, 1985.
5. H. Goldstein, Classical Mechanics, Narosa, 1990.
6. J. L. Synge and B. A. Griffith, Principles of Mechanics, McGraw-Hill, 1991.
7. L. N. Hand and J. D. Finch, Analytical Mechanics, Cambridge University Press, 1998.

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M.A./M.Sc. (FINAL) MATHEMATICS – 2021

Scheme of Examination : Annual Scheme

Note: 1. Papers I and II are compulsory

2. Candidates are required to opt any three papers from Paper III to XIII

COMPULSORY PAPERS

Paper – I: Analysis and Advanced Calculus

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Subspace of a metric space, Product space, Continuous mappings, Sequence in a metric space, Convergent, Cauchy sequence. Complete metric space, Baire's category theorem, compact sets, compact spaces, Separable metric space and connected metric spaces.

Unit 2: Normed linear spaces. Quotient space of normed linear spaces and its completeness. Banach spaces and examples. Bounded linear transformations. Normed linear space of bounded linear transformations. Weak convergence of a sequence of bounded linear transformations.

Unit 3. Equivalent norms, Basic properties of finite dimensional normed linear spaces and compactness. Reisz Lemma. Multilinear mapping. Open mapping theorem. Closed graph theorem. Uniform boundness theorem. Continuous linear functionals. Hahn-Banach theorem and its consequences. Embedding and Reflexivity of normed spaces. Dual spaces with examples.

Unit 4: Inner product spaces. Hilbert space and its properties. Cauchy-Schwartz inequality, Orthogonality and Functionals in Hilbert Spaces. Phythagorean theorem, Projection theorem, Separable Hilbert spaces and Examples, Orthonormal sets, Bessel's inequality, Complete orthonormal sets, Parseval's identity, Structure of a Hilbert space, Riesz representation theorem, Reflexivity of Hilbert spaces.

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Unit 5: Adjoint of an operator on a Hilbert space. Self-adjoint, Positive, Normal and Unitary operators and their properties. Projection on a Hilbert space. Invariance. Reducibility. Orthogonal projections. Eigen values and eigen vectors of an operator. Spectrum of an operator Spectral theorem.

Reference Books:

1. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons., 1978.
2. A. E. Taylor, Introduction to Functional Analysis, John Wiley, 1958.
3. W. Rudin, Functional Analysis, McGraw-Hill, 1973.

Paper II VISCOUS FLUID DYNAMICS

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration Theory Paper Max. Marks 100

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Viscosity , Analysis of stress and rate of strain, Stokes' law of friction, Thermal conductivity and generalized law of heat conduction, Equations of state and continuity , Navier- Stokes' equations of motion , Vorticity and circulation, Dynamical similarity, Inspection and dimensional analysis, Buckingham theorem and its application, Non-dimensional parameters and their physical importance : Reynolds number, Froude number, Mach number, Prandtl number, Eckart number, Grashoff number, Brinkmann number, Non – dimensional coefficients : Lift and drag coefficients, Skin friction , Nusselt number, Recovery factor.

Unit 2 : Exact solutions of Navier – Stokes' equations, Velocity distribution for plane couette flow, Plane Poiseuille flow, Generalized plane Couette flow, Hagen-Poiseuille flow, Flow in tubes of uniform cross-sections, Flow between two concentric rotating cylinders.

Unit 3 : Stagnation point flows : Hiemenz flow, Homann flow. Flow due to rotating disc, Concept of unsteady flow, Flow due to plane wall suddenly set in the motion (Stokes' first problem), Flow due to an oscillating plane wall (Stokes' second problem), Starting flow in plane Couette motion, Suction/injection through porous wall.

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Unit 4 : Equation of energy, Temperature distribution : Between parallel plates, in a pipe, between two concentric rotating cylinders, variable viscosity plane Couette flow, temperature distribution of plane Couette flow with transpiration cooling.

Unit 5 : Theory of very slow motion: Stokes' and Oseen's flows past a sphere, Concept of boundary layer , Derivation of velocity and thermal boundary equations in two-dimensional flow. Boundary layer on flat plate (Balsius-Topfer solution), Simple solution of thermal boundary layer equation for Pr = 1.

Reference Books:

1. J.L. Bansal, Viscous Fluid dynamics, JPH, Jaipur , 2008.
2. M.D.Raisinghania, Fluid Dynamics, S.Chand, 2003.
3. F. Chorlton, A Text Book of Fluid Dynamics, CBC, 1985.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, 1976.
5. S. I. Pai, Viscous Flow Theory I- Laminar Flow, D. Van Nostrand Co., Ing., Princeton, New Jersey, N.Y., Landon, Toronto, 1956.
6. F.M.White, Viscous Fluid Flow, McGraw-Hill, N.Y., 1974.

OPTIONAL PAPERS

Candidates are required to opt any three papers given below:

Paper – III: Continuum Mechanics

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Cartesian Tensors, Index notation and transformation laws of Cartesian tensors. Addition, Subtraction and Multiplication of cartesian tensors, Gradient of a scalar function, Divergence of a vector function and Curl of a vector function using the index notation. ε-δ identity. Conservative vector field and concept of a scalar potential function. Stokes', Gauss' and Green's theorems.

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Unit 2: Continuum approach, Classification of continuous media, Body forces and surface forces. Components of stress tensor, Force and Moment equations of equilibrium. Transformation law of stress tensor. Stress quadric. Principal stress and principal axes. Stress invariants and stress deviator. Maximum shearing stress.

Unit 3: Lagrangian and Eulerian description of deformation of flow. Comoving derivative, Velocity and Acceleration. Continuity equation. Strain tensors. Linear rotation tensor and rotation vector, Analysis of relative displacements. Geometrical meaning of the components of the linear strain tensor, Properties of linear strain tensors. Principal axes, Theory of linear strain. Linear strain components. Rate of strain tensors. The vorticity tensor. Rate of rotation vector and vorticity, Properties of the rate of strain tensor, Rate of cubical dilation.

Unit 4: Law of conservation of mass and Eulerian continuity equation. Reynolds transport theorem. Momentum integral theorem and equation of motion. Kinetic equation of state. First and the second law of thermodynamics and dissipation function. Applications (Linear elasticity and Fluids) – Assumptions and basic equations. Generalized Hook's law for an isotropic homogeneous solid.

Unit 5: Compatibility equations (Beltrami-Michell equations). Classification of types of problems in linear elasticity. Principle of superposition, Strain energy function, Uniqueness theorem, p-ρ relationship and work kinetic energy equation, Irrotational flow and Velocity potential, Kinetic equation of state and first law of Thermodynamics. Equation of continuity. Equations of motion. Vorticity-stream surfaces for inviscid flow, Bernoullis equations. Irrotational flow and velocity potential. Similarity parameters of fluid flow.

Reference Books:

1. W. Prager, Introduction to Mechanics of Continua, Lexinton Mass, Ginn, 1961.
2. A.C. Eringen, Mechanics of Continua, Wiley, 1967.
3. T.J. Chung, Continuum Mechanics, Prentice- Hall, 1988.

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Paper – IV: Boundary Layer Theory
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Derivation of boundary layer equations for two-dimensional flow. Boundary layer along a flat plate (Blasius-Topfer solution). Characteristic boundary layer parameters. Similar solutions. Exact solution of the steady state boundary layer equations in two-dimensional flow. Flow past a wedge. Flow along the wall of a convergent channel. Boundary layer separation. Flow past a symmetrically placed cylinder (Blasius series solution). Gortler new series method.

Unit 2: Plane free jet, Circular jet, Plane wall jet. Prandtl-Mises transformation and its application of plane free jet. Axially symmetrical boundary layers on bodies at rest. Boundary layers on a body of revolution. Mangler's transformation.

Unit 3: Three-dimensional boundary layers – Boundary layer flow on yawed cylinder. Growth of three-dimensional boundary layer on a rotating disc impulsively set in motion. Unsteady boundary layers – Method of successive approximations, Boundary layer growth after impulsive start of motion and in accelerated motion, Boundary layer for periodic flow (Pulsatile pressure gradient).

Unit 4: Approximate methods for the solution of the boundary layer equations. Karman momentum integral equation. Karman-Pohlhausen method and its application. Waltz-Thwaites method. Energy integral equation. Derivation of two-dimensional thermal boundary layer equation for flow over a plane wall.

Unit 5: Forced convection in a laminar boundary layer on a flat plate, Crocco's first and second integrals. Reynolds analogy. Temprature distribution in the spread of a jet – (i) Plane free jet, (ii) Circular jet (iii) Plane wall jet. Free convection from a heated vertical plate. Thermal-energy integral equation. Approximate solution of the Pohlhausan's problem of free convection from a heated vertical plate.

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Reference Books:

1. J.L. Bansal, Viscous Fluid dynamics, JPH, Jaipur , 2008.
2. R.K.Bansal, A Text Book of Fluid Mechanics, Laxmi publications Ltd., 2008.
3. F. Chorlton, A Text Book of Fluid Dynamics, CBC, 1985.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, 1976.
5. S. I. Pai, Viscous Flow Theory I- Laminar Flow, D. Van Nostrand Co., Ing., Princeton, New Jersey, N.Y., Landon, Toronto, 1956.
6. F.M.White, Viscous Fluid Flow, McGraw-Hill, N.Y., 1974.
7. H. Schlichting, Boundary Layer Theory, McGraw-Hill Book Co., N.Y,N.D.,1979.
8. K.Stewartson, The theory of laminar boundary layers in compressible fluids, Clarendon Press, Oxford, 1964.

Paper – V: Mathematical Programming
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Separating and supporting hyperplane theorems. Revised Simplex method for linear programming problem (LPP), Bounded variable problem. Convex function.

Unit 2: Integer programming. Gomory's algorithm for the all integer programming problem, Branch and Bound technique. Quadratic forms. Lagrange function and multiplier.

Unit 3: Non-linear programming problem (NLPP) and its fundamental ingredients, Necessary and Sufficient conditions for saddle points. Kuhn-Tucker theorem. Convex separable programming algorithm.

Unit 4: Kuhn-Tucker conditions for optimization for NLPP. Quadratic Programming, Wolf's method. Beale's method.Duality in quadratic Programming.

Unit 5: Dynamic programming, Principle of optimality due to Bellman, Solution of a LPP by dynamic programming.

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Reference Books:

1. Kanti Swaroop, P.K.Gupta and Manmohan, Operation Research, Sultan Chand & Sons., N.Delhi, 2007.
2. S.D.Sharma, Operations Research, Kedar Nath Ram Nath and co. Meerut, 2005.
3. F. S. Hillier and G. J. Lieberman, Introduction to Operations Research Concepts and Cases (9th Edition), Tata McGraw Hill, 2010.
4. Hamdy A. Taha, Operations Research, An Introduction (9th edition), Prentice-Hall, 2010.
5. G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 2002.

Paper – VI: Mathematical Theory of Statistics
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Sample space -Combination of events. Statistical independence, Conditional probability. Bay's repeated trials. Random variable, Distribution function, Probability function. Density function, Mathematical expectation, Generating function, Continuous probability distribution, Characteristic function. Fourier's inversion, Chebyshev and Kolmogrovea inequality. Weak and strong laws of large numbers.

Unit 2: Normal hypergeometric, rectangular, Negative Binomial Beta, Gamma and Cauchy's distribution. Methods of least squares and curve fitting, Correlation and Regression coefficients, Association of Attributes.

Unit 3: Interpolation– Introduction, Newton-Gregory theorem. Newton's, Lagrange's, Gauss's and Striling's formulae.

Index numbers– Introduction, Price relatives, Quantity relatives, Value relatives, Link and Chain relatives. Aggregate methods, Fisher's ideal Index. Change of the base of the index numbers. Elementary sampling theory. Distribution of means of samples for Binomial. Cauchy, rectangular and normal population. Exact distributions of χ^2, t, z and F. Statistics in samples from a normal population, their simple properties and applications.

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Unit 4: Test of significance and difference between two means and two standard deviations for large samples with modification for small samples and taken from normal population. Analysis of variance, Simple cases (One criteria and two criteria of classification).

Unit 5: Elementary Statistical theory of Estimation of efficient, Fisher's criteria for the estimator, Consistant, Efficient and Sufficient estimator, Method of maximum likelihood. Maximum Likelihood Estimator, Other methods of estimation. Methods of moments, Minimum variance, Minimum Chi-square and Least Squares.

Reference Books:

1. S.C. Gupta and V.K.Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Sons., N. Delhi, 2008.
2. J.N.Kapur and H.C.Saxena, Mathematical Statistics, S.Chand, 2010.
3. S.M. Ross, Introduction to Probability Models (Sixth edition) Academic Press, 1997.
4. Blake, An Introduction to Applied Probability, John Wiley & Sons, 1979.
5. J. Pitman, Probability, Narosa, 1993.
6. A.M. Yagolam and I.M. Yagolam, Probability and Information, Hindustan Publishing Corporation, Delhi, 1983.

Paper – VII: Combinatorics and Graph Theory
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Introduction to diagraph, Orientation of a graph, Underlying graph, Parallel edges, Source and Sink, Types of digraphs, Accessibility, Arborescence, Spanning arborescence, Euler digraphs, Handshaking dilemma, Incidence matrix of a digraph, Circuit matrix of a digraph. Degree sequences, Graphic sequence, Havel Hakimi Theorem.

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Unit 2: Matrix representation of graphs except adjacency and incidence matrix.
Planar graphs Kurotowski's graphs, Maximal planar graphs, Outer planar graphs, Maximal outer planar graph, Minimally non-outer planar graph, Thikness and Crossing number of bipartite and complete bipartite graph, Euler's formula, Kuratowski's theorem. Isomorphism, Homomorphism.

Unit 3: Graph theory in Network analysis Network flows, Transport networks, Max-flow min-cut-theorem. Cut set and Cut vertices, Cut set and bridge, fundamental cut sets, Connectivity and Severability, Vector spaces of graphs.

Unit 4: Enumeration of graphs:
Types of enumeration, Labeled graphs, Counting labeled trees, Rooted labeled trees, Enumeration of graphs, Partitions, Generating functions, Counting unlabeled trees, Rooted unlabeled trees, Permutation, Composition of permutation, Pólya's theorem, Burnside's lemma, Pólya's enumeration theorem

Unit 5: Coloring: Graph coloring, Vertex coloring, Edge coloring, Properly coloring of a graph, Chromatic polynomial, Decomposition theorem, Four colour theorem, The five colour theorem.
Graph and Algorithms: Applications, Shortest path algorithms, Dijkstra's algorithm, Algorithm for minimal spanning tree, Kruskal's algorithm, Prim's algorithm, The labeling algorithm

Reference Books:

1. K.H. Rosen, Discrete Mathematics and it's Applications, McGraw Hill, 1999.
2. C.Vasudev, Graph theory and it application, New Age International Pvt., 2006.
3. N.L. Biggs, Discrete Mathematics, Oxford Science Publication, 1985.
4. T. Koshy, Discrete Mathematics with Applications, Academic Press, 2005.
5. N. Deo, Graph Theory, Prentice Hall of India, New Delhi, 2004.

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Paper- VIII: Integral Transforms and Integral Equations
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Laplace transform– Definition and its properties. Rules of manipulation. Laplace transform of derivatives and integrals. Properties of inverse Laplace transform. Convolution theorem. Complex inversion formula.

Unit 2: Fourier transform – Defiition and properties of Fourier sine, cosine and complex transforms. Convolution theorem. Inversion theorems. Fourier transform of derivatives. Mellin transform– Definition and elementary properties. Mellin transforms of derivatives and integrals. Inversion theorem. Convolution theorem.

Unit 3: Infinite Hankel transform– Definition and elementary properties. Hankel transform of derivatives. Inversion theorem. Parseval Theorem.
Solution of ordinary differential equations with constant and variable coefficients by Laplace transform. Application to the solution of Simple boundary value problems by Laplace, Fourier and infinite Hankel transforms.

Unit 4: Linear integral equations– Definition and classifications. Conversion of initial and boundary value problems to an integral equation. Eigen values and Eigen functions. Solution of homogeneous and general Fredholm integral equations of second kind with separable kernels Solution of Fredholm and Volterra integral equations of second kind by methods of successive substitutions and successive approximations. Resolvent kernel and its results. Conditions of uniform convergence and uniqueness of series solution.

Unit 5: Solution of Volterra integral equations of second kind with convolution type kernels by Laplace transform. Solution of singular integral equations by Fourier transform.

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Integral equations with symmetric kernels– Orthogonal system of functions. Fundamental properties of eigen values and eigen functions for symmetric kernels. Expansion in eigen functions and bilinear form. Hilbert-Schmidt theorem. Solution of Fredholm integral equations of second kind by using Hilbert-Schmidt theorem. Classical Fredholm theory– Fredholm theorems. Solution of Fredholm integral equation of second kind by using Fredholm first theorem.

Reference Books:

1. Shanti Swarup, Integral Equations, Krishna Publications, Meerut.
2. M.D.Raisinghania, Integral Equations and Boundary Value Problems, S.Chand, 2010.
3. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and their Applications, Taylor and Francis Group, 2014.
4. Abdul J. Jerry, Introduction to Integral Equations with applications, Marcel Dekkar Inc. NY, 1999.
5. L.G.Chambers, Integral Equations: A short Course, Int. Text Book Company Ltd. 1976.
6. Murry R. Spiegal, Laplace Transform (SCHAUM Outline Series), McGraw-Hill, 1965.

Paper- IX: Relativity and Cosmology

Teaching : 6 Hours per Week

Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Relative Character of space and time, Principle of Relativity and its postulates, Derivation of special Lorentz transformation equations, Composition of Parallel velocities, Lorentz-Fitzgerald contraction formula, Time dilation, Simultaneity, Relativistic transformation formulae for velocity, Lorentz contraction factor, Particle acceleration, Velocity of light as fundamental velocity, Relativistic aberration and its deduction to Newtonian theory.

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Unit 2: Variation of mass with velocity, Equivalence of mass and energy, Transformation formulae for mass, Momentum and energy, Problems on conservation of mass, Momentum and energy, Relativistic Lagrangian and Hamiltonian, Minkowski space, Space-like, Time-like and Light-like intervals, Null cone, Relativity and Causality, Proper time, World line of a particle.

Unit 3: Principles of Equivalence and General Covariance, Geodesic postulate, Mach's principle, Newtonian approximation of equation of motion, Einstein's field equation for matter and empty space, Reduction of Einstein's field equation to Poisson's equation, Schwarzschild exterior metric, its isotropic form and singularity, Relativistic differential equation for orbit of the planet.

Unit 4: Three crucial tests in general Relativity and their detailed descriptions, Analogues of Kepler's laws in General Relativity, Trace of Einstein tensor and energy-momentum tensor for perfect fluid, proof of its expression for perfect fluid, Schwarzs child interior metric and boundary conditions, Radar Echodelay (Fourth test).

Unit 5: Lorentz invariance of Maxwell's equations and their tensor form, Lorentz force on charged particle, Energy-momentum tensor for electromagnetic field, Reissner-Nordstrom metric for spherically charged particle.
Cosmology – Einstein's field equation with cosmological term, static cosmological models (Einstein and de-Sitter) and their physical and geometrical properties. Red Shift in non-static form of de-Sitter line-element. Einstein-space, Hubble's law, Weyl's postulate.

Reference Books:

1. J.V. Narlikar, Lectures on General Relativity and Cosmology, Macmillan Co. Ltd. India, N.Delhi, 1978.
2. C. Moller, The Theory of Relativity, Oxford Clarendon Press, 1952.
3. P.G. Bergmann, Introduction to the Theory of Relativity, Prentice Hall of India, 1969.
4. J.L. Anderson, Principles of Relativity Physics, Academic Press, 1967.
5. W. Rindler, Essential Relativity, Van Nostrand Reinhold Company, 1969.
6. V. A. Ugarov, Special Theory of Relativity, Mir Publishers, 1979.

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Paper – X: Industrial Mathematics
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Partial differential equations and techniques of solution. Finite difference methods for solving PDE. Application to problems of industry with special reference to Fluid Mechanics. Operational Techniques.

Unit 2: Operational Techniques. Computational procedure of Simplex method, Two-phase Simplex method, Revised Simplex method, Duality, duel simplex method.

Unit 3: Sensitivity Analysis in Linear Programming Problems, Various models of Assignment problems, alternate optimal solutions, post optimality analysis in transportation.

Unit 4: Inventory Models. EOQ models with and without shortages. EOQ models with constraints.

Unit 5: Replacement and Reliability models. Replacement of items that deteriorate, Replacement of items that fail completely.
Reliability Theory – Coherent structure, Reliability of systems of independent components, Bounds on system reliability, Shapes of the system reliability function, Motion of aging, Parametric families of life distribute with Monotone failure rate.

Reference Books:

1. Kanti Swaroop, P.K.Gupta and Manmohan, Operation Research, Sultan Chand & Sons., N.Delhi, 2007.
2. S.D.Sharma, Operations Research, Kedar Nath Ram Nath and co. Meerut, 2005.
3. H.A.Taha, Operations Research: An Introduction; MacMillan Publishing Company, New York, 1982.
4. F.S. Hillier and G.J. Leiberman, Introduction to Operations Research; Holden Day, 1962.
5. I.N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1988.

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Paper – XI: Magnetohydrodynamics
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Maxwell electromagnetic field equations. Constitutive equations of fluid motion, Stokes hypothesis. Maxwell stress tensor. Fundamental equations of Magnetofluid-dynamics. Magnetofluiddynamic approximations. Magnetic field equation, Frozen in fluid, Alfven transverse waves. MHD boundary conditions. Inspection and Dimensional analysis, π-products.

Unit 2: Reynolds number, Mach number, Prandtl number, Magnetic Reynolds number, Magnetic pressure number, Hartmann number, Magnetic parameter, Magnetic Prandtl number and Nusselt number. Hartmann plane Poiseuille flow and plane Couette flow including temperature distribution. MHD flow in a tube of rectangular cross-section. MHD pipe flow. MHD flow in annular channel. MHD flow between two rotating coaxial cylinders.

Unit 3: MHD flow near a stagnation point. MHD Reyleigh's flow. MHD Stoke's flow past a sphere, MHD Oseen's flow past a sphere. MHD boundary layer flow past a flat plate in an aligned magnetic flow. Wilson's numerical solution technique.

Unit 4: MHD boundary layer flow past a flat plate in a transverse magnetic field. modified Rossow's method of solution. MHD plane free jet flow. Wave and theory of characteristics, Equation of the characteristics, Characteristic surfaces, MHD characteristic equations. MHD waves.

Unit 5: Friedriches diagrams. Dispersion relation. MHD shock waves. Generalized Hugoniot condition. Compressive nature of MHD shocks. MHD shock wave classification. MHD shock stability.

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Reference Books:

1. J.L.Bansal, Magnetofluiddynamics of Viscous Fluids, JPH, Jaipur, 1994.
2. K.R. Cramer and S.I.Pai, Magnetofluiddynamics for Enginners and Applied Physicists, McGraw-Hill, N.Y., 1973.
3. P.A. Davidson, An Introduction to Magnetohydrodynamics, Cambridge Univ. Press, U.K., 2001.
4. J.A.Shercliff, A Textbook of Magnetohydrodynamics, Pergamon Press., 1965.
5. K.R. Cramer and S.I. Pai, Magnetofluid Dynamics for Engineers and Applied Physicists, McGraw- Hill Book Co., 1973.

Paper- XII: Advanced Numerical Analysis
Teaching : 6 Hours per Week
Examination : Common for Regular/Non-collegiate Candidates

3 Hrs. duration **Theory Paper** **Max. Marks 100**

Note : This paper is divided into FIVE Units. TWO questions will be set from each Unit. Candidates are required to attempt FIVE questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit 1: Iterative methods – Theory of iteration method, Acceleration of the convergence, Chebyshev method, Muler's method, Methods for multiple and complex roots. Newton-Raphson method for simultaneous equations, Convergence of iteration process in the case of several unknowns.

Unit 2: Solution of polynomial equations – Polynomial equation, Real and complex roots, Synthetic division, the Birge-Vieta, Bairstow and Graeffe's root squaring method. System of simultaneous Equations (Linear)- Direct method, Method of determinant, Gauss-Jordan, LU-Factorizations-Doolitte's, Crout's and Cholesky's. Partition method. Method of successive approximate-conjugate gradient and relaxation methods.

Unit 3: Eigen value problems– Basic properties of eigen values and eigen vector, Power methods, Method for finding all eigen values of a matrix. Jacobi, Givens' and Rutishauser method. Complex eigen values.

Curve Fitting and Function Approximations – Least square error criterion. Linear regression. Polynomial fitting and other curve fittings, Approximation of functions by Taylor series and Chebyshev polynomials.

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Unit 4: Numerical solution of Ordinary differential Equations – Taylor series Mathod, Picard method, Runge-Kutta methods upto fourth order, Multistep method (Predictor-corrector strategies), Stability analysis – Single and Multistep methods.

Unit 5: BVP's of ordinary differential Equations – Boundary value problems (BVP's), Shooting methods, Finite difference methods. Difference schemes for linear boundary value problems of the type $y'' = f(x,y)$, $y'' = f(x,y,y')$ and $y^{iv} = f(x,y)$.

Reference Books:

1. S.S.Sastry, Introductory Methods of Numerical Analysis, PHI, 1979.
2. V.Rajaraman, Computer Oriented Numerical Methods, PHI, 1993.
3. M.K.Jain, S.R.K. Eyenger and R.K. Jain, Numerical Methods for Mathematics and Applied Physicists, Wiley-Eastern Pub., N.Delhi, 2005.
4. B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
5. C. F. Gerald and P. O. Wheatley, App;ied Numerical Analysis, Pearson Education, India,7th edition, 2008.
6. C.F. Gerald, P.O. Wheatley, Applied Numerical Analysis, Addison-Wesley, 1998.
7. S. D. Conte, C de Boor, Elementary Numerical Analysis, McGraw-Hill, 1980.
8. C.E. Froberg, Introduction to Numerical Analysis, (Second Edition), Addition-Wesley, 1979.

Paper – XIII: Computer Applications

Teaching: 4 Hours per Week for Theory Paper.

Examination: For Regular candidates only.

3 Hrs.duration	**Theory Paper**	**Max.Marks 70**
2 Hrs. duration	**Practical**	**Max. Marks. 30**

Note: This paper is divided into **Five** Units. TWO questions will be set from each Unit. Candidates are required to attempt **Five** questions in all taking ONE question from each Unit. All questions carry equal marks.

Unit – 1

Computer languages, System software and application software. Windows: Graphical user interface, control panel and all features there in files and folders management. Using Accessories, Getting help, copying, moving and sharing information between programs. Setting up printer and fonts. Configuring modem.

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Unit 2:

Introduction to MS Word and Ms-Excel. Algorithms and flow charts. Programming languages and problem solving on computers.

Unit 3

Arithmetic expressions, Input-output, Conditional statements, Implementing loops in programs. Defining and manipulating arrays,

Unit 4.

Programming using Matlab/Mathematica/Maple – Variables, Vector and Matrix Computation, Built-in-functions, Plotting, output, M-files.

Unit 5

Programming using Matlab/Mathematica/Maple – Functions, Loops, Conditional Execution, Matrix Multiplication.

Reference Books:

1. Y. Kanetkar, Let Us C, BPB Publications, 2008.
2. C. Ghezzi and M. Jazayeri, Programming Languages Concepts, John Wiley, 1977.
3. M. Marcotty & H.F. Ledgard, Programming Language Landscape, Galgotia Publication, 1981.
4. R.C. Hutchinson and S.B. Just, Programming using the C Language, McGraw-Hill.
5. John H. Mathews and Kurtis D. Fink, Numerical Methods using MATLAB, PHI, N.Delhi, 1999.
6. Brian R. Hunt, Ronald L. Lipsman, Jonathan M. Rosenberg, A Guide to MATLAB, Cambidge Univ. Press, 2001.
7. Duane hanselman and Bruce Littlefield, Mastering Matlab-7, Pearson Education 2005.
8. William J. Palm III, Introduction to Matlab-7 for Engineers, McGraw Hill, 2005.
9. Mureşan, Marian, Introduction to Mathematica® with Applications, Springer, 2017.
10. José Guillermo Sánchez León, Mathematica Beyond Mathematics: The Wolfram Language in the Real World, Chapman and Hall/CRC, 2017.

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Practical

Teaching: 2 Hours per week per batch not more than 20 students

Examination: 2 Hours duration **Max.Marsk: 30**

Solution of system of linear equations – Gauss elimination, Gauss-Seidel, Eigenvalues and Eigenvectors – Power method and inverse power method. Least Squares Approximation – Fitting of straight line, parabola and cubic equation. Numerical integration – Trapezoidal and Simpson's methods, Differential equations and graphics: Double integration, Roots of polynomial, two and three dimensional plots, Numerical solution of Initial value problems – Euler's method, Fourth order Runge-Kutta method, solution of Boundary value problems by using inbuilt functions of MATLAB/Mathematica/Maple/Scilab.

Distribution of Marks:

Two Practical – 10 Marks each	= 20 Marks
Practical Record	= 05 Marks
Viva-Voce	= 05 Marks
Total Marks	= 30 Marks

Note:

1. Problems will be solved by using MATLAB/Mathematica/Maple/Scilab.
2. Each candidate is required to appear in the Practical examination to be conducted by internal and external examiners. External examiner will be appointed by the University through BOS and internal examiner will be appointed by the Head of the Department/Principal of the College.
3. Each candidate has to prepare his/her practical record.
4. Each candidate has to pass in Theory and Practical examinations separately.

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